Mathematical morphology, granulometries and texture perception

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ABSTRACT

In this work we present a classical morphological tool, granulometry, and a practical application on medical images, pneumoconiosis classification. The radiologist diagnose on these images is based on a preattentive discrimination process of the textural patterns appearing at the pulmonar parenchyma. Thus, in order to automatize this classification we have chosen a tool which agrees with perceptual theories of Computer Vision on texture discrimination. Our work is centered, concretely, on the perceptual models based on texton theory. These works base texture discrimination on differences in density of texton attributes. We link this approach with a morphological tool, granulometry, as a helpful multi-scale analysis of image particles. The granulometric measure provides a density function of a given feature, which depends on the family of algebraic openings selected. Thus in this paper we define different granulometries which allow us to measure the main texton features, such as, shape, size, orientation or contrast, proposing a granulometric analysis as a systematic tool for texture discrimination according to a perceptual theory. And finally, we present the application of measuring size density on some radiographic images suffering from pneumoconiosis.

1. INTRODUCTION

Texture discrimination has become a classical problem in computer vision, in this work we show an application of this field on medical images. Pneumoconiosis is a disease caused by a constant inhalation of particles, usually suffered by workers in permanentely dusty atmospheric conditions. Chest radiographies are the main tool in order to diagnose pneumoconiosis disease, which present a textural pattern formed by bright opacities. An experimented radiologist is able to classify a large number of images in a short time. In view of these considerations we can suppose that the classification process is based on a preattentive perception process. With the goal of getting an automatization of the process we will center our work in perceptual models, in particular we will chose the frame of the texton theory, and we will link it with a classical morphological tool. Let us briefly summarize the main goals of this article.

First of all we present a brief review of works on texture discrimination. We draw our attention to those works which intend to be a general model for preattentive texture discrimination which focus their attention in obtaining results which agree with human visual perception, in front of those works based on the mathematical modelization of textures. In the first case we mainly start from works following texton theory based on differences in the density of textons attributes.

Secondly we introduce a classical tool in Mathematical Morphology, the granulometry. The axioms given by Matheron,[22] to formalize a granulometry, deal with a multi-scale image filtering analysis.[19,9,17] The multi-scale approach has provided interesting results in various fields of computer vision. The granulometric measurement provides a density function of a given feature, which depends on the family of algebraic openings selected to perform the multi-scale analysis. Consequently we define different granulometries which will allow us to measure the main features of image particles. We show some examples on natural images taken from Brodatz's album.[2]

Finally we present the results obtained from applying the previous ideas on radiographic images suffering from pneumoconiosis. We show different results from different algebraic openings families.

2. PERCEPTUAL TEXTURE DISCRIMINATION: A BRIEF REVIEW

Texture discrimination is an important area in Computer Vision. An important treatment of this topic is the one based on the use of mathematical models capable of describing and, normally, to synthesize a textured image, considering image texture as a particular result of that model.[4] In this sense Mathematical Morphology has provided outstanding models as the Boolean model or the Dead Leaves Functions models.[11,23]

The second approach uses image measurements based on perceptual considerations. Early works tried to define measures whose behaviour agreed with the perceptual description of a textural property.[10,28] Among these methods we can find the statistical approach, oriented to statistical properties at the pixel level, and the structural approach aimed to extract structures or region features of images.[8] The psycho-physical studies performed by Julesz[12,15,14,13] culminated in the well-known texton theory — which agrees with the Marr's Primal Sketch.[20,21] Recently a general computational model based on perceptual considerations has been presented by J. Malik and P. Perona.[18] This model intends to be consistent with physiological mechanisms of early vision and its results match psychophysical data. In all cases it is assumed that pre-attentive[*] texture discrimination depends on differences in the density of textons, called blobs by Marr, as well as on their attributes — orientation, shape, size or contrast. These blobs are regarded as image regions which are either brighter or darker than the background. These have been defined as the duals of edges by H. Voorhees and T. Poggio,[30,31] or the regions associated with a — at least one — local extremum point by T. Lindeberg.[17]

Some works treat the problem of measuring the density of blobs attributes. H. Voorhees and T. Poggio[31] propose a general blob detection and a posterior attribute measurement for each detected blob. Once the measurements are made, we obtain the distributions of attributes which can be compared in order to discriminate. Another important contribution comes from R. Vistnes[29] who proposes computing attribute distributions without isolating image substructures, or blobs, provided that the existence of such image structures is uncertain in a statistical sense. Thus he proposes to test statistically the hypothesis that a structure with a particular feature value exists in a image, then to estimate a feature histogram is made by combining different attributes values. In other words, to estimate histograms of edge orientation he would compute orientation detectors in several directions.

Out of this last point of wiew we propose some morphological tools to obtain approximations of blob attributes densities. Heretofore we will use the term particle, more common in morphological processing, in the same sense as blob or texton.

3. GRANULOMETRY

A granulometry is a generic method based on a sieving process used to calculate size distributions of particles of certain materials. Matheron's formalization[22] allowed its use in image analysis. He defined the axioms to be accomplished by a family of transformations to suit granulometry calculation:

Definition 3..1 *A family of parametric transformations $\{\phi_\lambda\}$ in $\lambda > 0$ allows calculating a granulometry if*

1. $\forall \lambda > 0,\ \phi_\lambda$ *is an algebraic opening,*
2. *Stability of the parameters is acccomplished*

$$\phi_\lambda \circ \phi_\mu = \phi_\mu \circ \phi_\lambda = \phi_{Sup(\mu,\lambda)} \ \ \forall \lambda, \mu \geq 0 \tag{1}$$

where an algebraic opening is defined as:

Definition 3..2 *An algebraic opening is any mapping $\phi : \mathcal{F} \longrightarrow \mathcal{F}$ fulfilling the following axioms:*

(i) Antiextensivity

$$\phi(f) < f \tag{2}$$

[*] Differences between attentive and pre-attentive vision are fairly well explained by Julesz[12]

(ii) Preservation of order, or increase

$$f < g \Longrightarrow \phi(f) < \phi(g) \tag{3}$$

(iii) Idempotence

$$\phi[\phi(f)] = \phi(f) \tag{4}$$

Serra and Vincent's work[27] presents a non-exhaustive catalogue of openings. There are four main types of openings: morphological openings, trivial openings, connected openings and envelope openings. From these types and by cross-union of various types, we can define a large number of different algebraic openings.

Considering $f(x, y)$ and $g(x, y)$ as finite-support greytone image functions, defined in $\mathbb{Z}^2$ and ordered by the following relation

$$f < g \Leftrightarrow f(x, y) < g(x, y) \quad \forall (x, y) \tag{5}$$

we can construct several families of transformations suitable to calculate a granulometry on an image.

In short, a granulometry is based essentially on a sieving process and provides a size distribution of particles. Nevertheless, we are working on abstract sieves, which not only provide information on size but also on other features of particles. One of the goals of this work was to link the granulometry to first order statistics[†] of textural features. So, we propose to interpretate what we measure on particles as texton attributes.

To calculate a granulometry we firstly have to describe the opening, normally depending on a parameter b — the structuring element if the opening is based on morphological transformations –, and the parameters that define the transformation family. This selection depends on the blob feature measured.

Once we have applied the transformations to the image, one must calculate the granulometric curve, we will name it $GC_f(p, b(p))$. This curve is a feature distribution function:

$$GC_f(p, b(p)) = \frac{1}{\eta} \cdot \frac{dM(\phi_p(f))}{dp} \quad p \geq 0 \tag{6}$$

where M represents a measure, which can be any Lebesgue measure, and η corresponds to a normalization parameter, in such a way that

$$\int_0^\infty GC_f(p, b) dp = 1 \tag{7}$$

A discrete formulation of the expression above, assuming finite difference as a derivative approximation, is obtained by

$$GC_f(p, b) = \frac{1}{\eta} \cdot (M(f) - M(\phi_p(f))) \quad p \geq 0 \tag{8}$$

where $\eta = M(f) - M(\phi_{p_{max}}(f))$.

Once we have defined the granulometry, next, we will show how can we use the granulometric curve as an approximation of density for texton attributes.

[†]The term *first order statistics* of a feature is used by Julesz as a density of the feature on the image

4. A GENERAL TOOL FOR PERCEPTUAL TEXTURE DISCRIMINATION

Once we have introduced the granulometry tool we want to relate it to the perceptual theories showed in section 2. In order to obtain a texture discrimination which agrees with human results, we have to find some functions presenting the distribution of texton features on the image, such as the probability distribution to find a particle of a given size, orientation, shape or contrast. Granulometry, as defined‡ in the previous section, does not provide a methodology to detect isolated particles and measuring them afterwards, however it can be a useful tool in order to compute feature distribution approximations without the previous isolating process, in the same sense as Vistnes' work.[29]

4.1. Size

By definition, a granulometry is a generic method to calculate size distribution of particles. By computing a granulometry on an image we obtain an approximation of the probability to see a particle of a given size in the image. It is not easy to define the meaning of size as J. Serra[25] states. In our case we constrained the concept to individualized particles. Here particle size is very related to its shape, that is, given a particle shape one can easily find a good size descriptor for it.

If we know the a priori shape of particles, we can define a granulometry using a family of morfological openings with a structuring element accordingly with the particles shape. The structuring element $b(p)$ has an associated parameter p to define the size or scale of b.

In the event of not knowing the shape of the particle, we can use the inside longest straight distance between any two points. We can obtain this measure computing a granulometry by a family of algebraic openings defined as the supremum of morphological openings with linear structuring elements in the main directions, these are denoted by B_θ

$$\begin{aligned} \phi_p(f) &= \sup_\theta((f \ominus b(p)) \oplus b(p)) = \\ &= \sup_\theta((f \ominus pB_\theta) \oplus pB_\theta) \end{aligned} \tag{9}$$

In figure 3 we show some results of applying this algebraic opening§ on the chest images suffering from pneumoconiosis, as we will see in a posterior section this is a clear case of texture classification based on particle size.

4.2. Shape

Measuring or describing shape of particles is an important problem. P. Maragos studied it in depth[19] and defined a shape-size descriptor called *pattern spectrum*. He also demonstrated the ability of the pattern spectrum to measure shape-size relation.

The pattern spectrum is given by

$$PS_f(r,g) = -\frac{dA((f \ominus rg) \oplus rg)}{dr} \quad r \geq 0 \tag{10}$$

hence we can directly relate it to a granulometric curve

$$PS_f(r,g) = -\eta \cdot GC_f(r, b(r)) \tag{11}$$

‡M. Coster and J. Chermant[6] distinguish two types of granulometries, in *number* and in *measure*. In our case we have defined a measure granulometry. A mumber granulometry is defined by a family of algebraic openings which can isolate the particles and follows with a counting process in spite of a measuring process.

§J. Serra[26] demonstrates that the supremum of a family of algebraic openings is also an algebraic opening

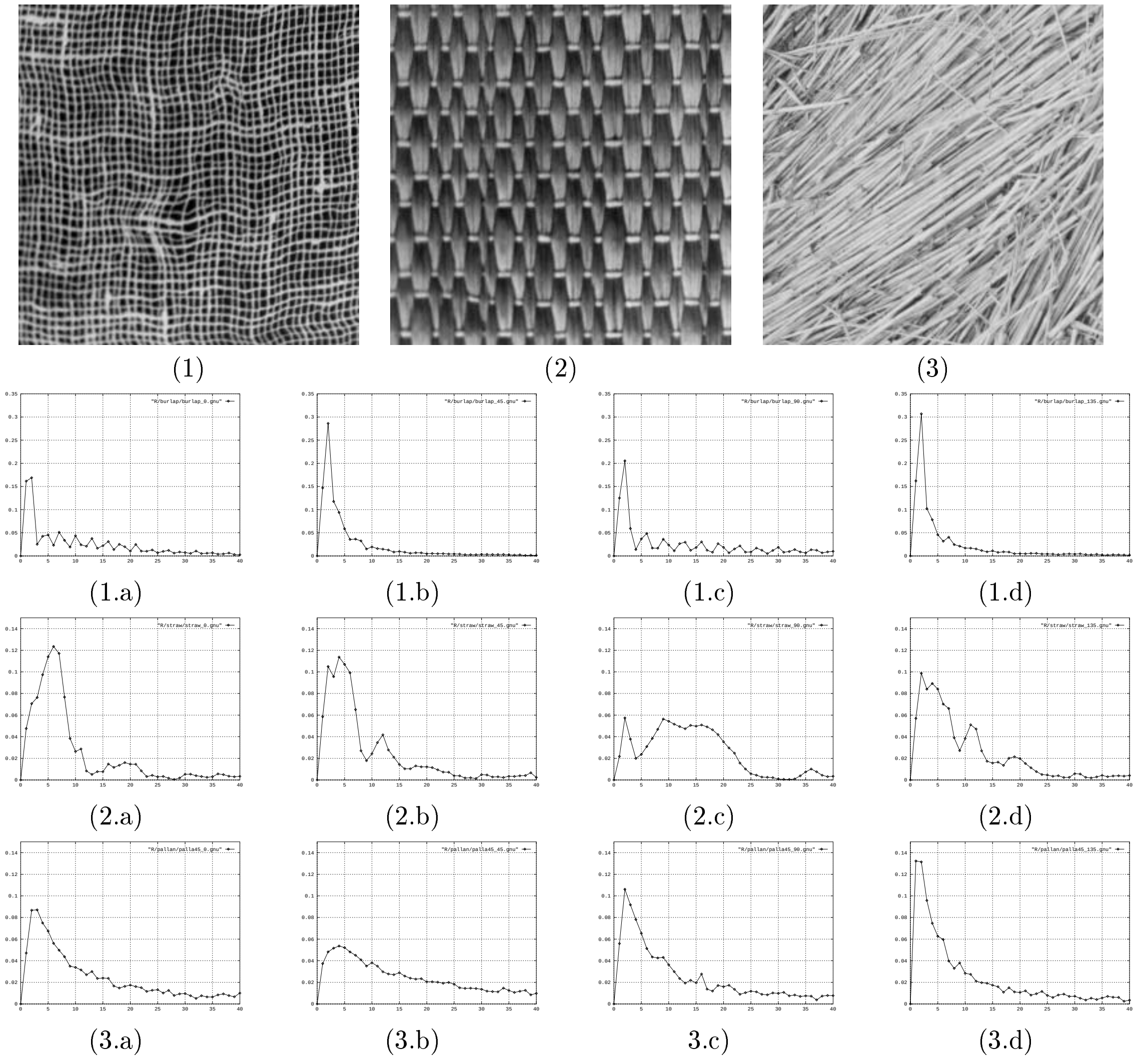


Figure 1: (1), (2) and (3) are the original images. (1.a), (1.b), (1.c) and (1.d) are the granulometric curves with a linear structuring element in a given direction $0°, 45°, 90°$ and $135°$, respectively, for image 1.

where $b(r) = r \cdot g$ and

$$M(f) = A(f) = \int_{\mathbf{R}^m} f(x)dx \tag{12}$$

Therefore the morphological opening has proved to be a good opening to describe shape. In this case $b(r)$ represents a structuring element of a given shape and scale r, it will determine the behaviour of the morphological opening with respect to the shape of the image particles.

As we have seen either PS as GS are functions depending on two parameters: scale r, and function $b(r)$. By fixing this last one we have a classical granulometric curve for a given structuring element, but varying both r and $b(r)$ we obtain the called *full pattern spectrum*,[19] as a *complete shape-size descriptor*.

In short, a set of granulometries can be a good complete descriptor of shape. We will apply these concepts to other specific granulometries in order to obtain *complete feature-size descriptors*.

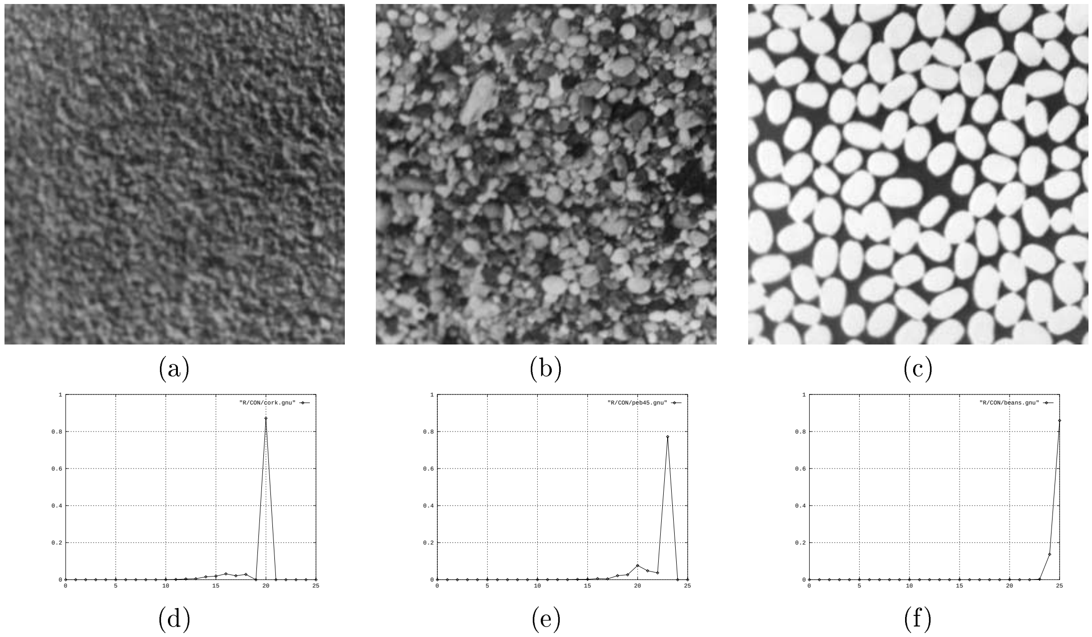


Figure 2: (a), (b) and (c) are the original images. (d), (e) and (f) are the three corrresponding granulometric curves.

4.3. Orientation

Defining the orientation of a given blob consists in determining how the blob lies in the field of view, we have to assume that it is an elongated blob[¶]. Thus the orientation of the object is defined by the orientation of the *axis of elongation*. Usually it has been computed choosing the axis of least second moment.[1] To identify a particular line in an image we need to specify an angle and a distance from a x-y coordinate system.

Considering that we do not detect the blob, we will measure the distribution of orientation directly on the image — following the Vistnes's model. Using a granulometry we can compute the probability to find a particle of any size with an axis of elongation in a given orientation. It can be obtained by defining a family of morphological openings with a linear structuring element in a given direction θ, as B_θ defined in section 4.1.

Just as we need a x-y coordinate system to describe orientation, similarly we will need more than one granulometric curve for a complete orientation description. Since each of them describe the orientation-size relation for a given direction, we define a *complete orientation-size descriptor* by varying the θ parameter, in the same sense that Maragos defines the *full pattern spectrum*. Therefore, a set of granulometries differing in θ parameter can provide the probability of finding a particle in a given direction with regard to its scale or size. From this standpoint we show in figure 1 some images where a group of four granulometries have been measured, in order to give a full description of the predominant orientation of image particles. All of them present a global maximum at the first sizes, but the differences of their maximum values depend on the predominant orientation of image blobs. Therefore we can say that the smaller values of these maxima indicate which is the predominant orientation.

4.4. Contrast

It has been demostrated[31] that the constrast of a blob is a good blob attribute for texture discrimination. A review of the literature reveals that obtaining an approximation for the density of contrast values of blobs, without a previous blob detection, is a non trivial problem.

The foregoing problem is illustraded by M. Grimaud.[7] He presents an important reviewing of the classical

[¶]Elongated blobs have been named *bars* by Marr primitives for contrast measurement as Rh-maxima or h-maxima, showing their main problems. He shows the weakness of the Rh-maxima in front of noise, and the non stability of parameters in a family of h-maxima transformations. Nevertheless, in the same work he offers a new measure, the dynamic of the extrema which allows to work in terms of contrast without regard to the size or shape of the substructures. It would be an interesting measure in order to construct a family of transformations, for granulometric purposes, based on a geodesic reconstruction of extrema with a dynamic value greater than a given h. However these transformation do not satisfy the increasing property, which is required to be suitable for a granulometry.

In view of these considerations we have not a good algebraic opening for contrast measuring. Even though, we show the granulometric curves obtained from applying a family of algebraic openings constructed by a geodesic reconstruction of image maxima which are greater than a h value (see figure 2). Consequently we obtain a density distribution of the intensity value of the image maxima. Which is a measurement at the pixel level, and does not supply us with enough information of the particles as image substructures.

5. PRACTICAL APPLICATION: PNEUMOCONIOSIS CLASSIFICATION

As we have explained in the introduction, we want to automate the classification process for a set of radiographic images suffering from pneumoconiosis. This disease manifests itself as small rounded opacities appearing on the radiographs and forming a certain textural pattern of bright blobs. The radiologist classifies the images according to a size criterion of the opacities — large, medium or small. They may classify a large number of images in a short time, thus this seems to be a preattentive perception problem. This problem has been widely studied from different points of view.[16,24,3] Here we show this problem as an automatic classification, where textures differs in values of size densities of image blobs — bright opacities. In figure 3 we can see a group of three images presenting different sizes. Then we present the resulting granulometric curves from applying three different families of algebraic openings, where we can see a maximum at the predominant size of opacities in the image:

1. *Morphological openings with rounded structuring elements.* Their granulometric curves are given in graphics a.1, b.1, c.1. These curves provide clear information on size distribution of rounded opacities, with maximum values inside the corresponding size interval.

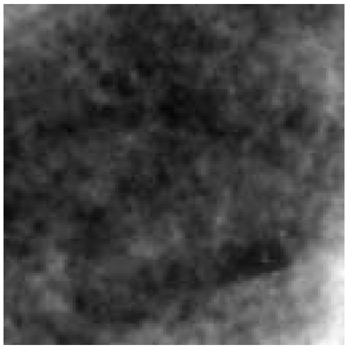
2. *Erosion with rounded structuring elements followed by reconstruction.* Curves are given in graphics a.2, b.2 an c.2. In this case we obtain a good classification for large opacities. However, the smaller opacities are not well classified due to the reconstruction transformation, which connect neighbouring overlapped opacities.

3. *Supremum of morphological openings with linear structuring elements in different directions* ($\theta = 0°, 45°, 90°, 135°$). Granulometric curves are a.3, b.3 and c.3. The conclusion we can draw from these curves is the same as in the first case, but curves are smoother. This is due to robustness of this opening at detecting the main diameter of the opacity.

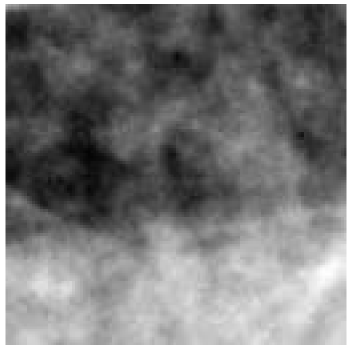
6. CONCLUDING REMARKS

In this work we have given a parcial solution to the pneumoconiosis classification problem, placing it in the frame of perceptual texture discrimination. The problem has not been completely resolved because there are some disease stages which do not form clear textural patterns. For this reason and in order to give a homogeneus treatment to all images, we bring up the necessity to use more refined techniques which isolate individual particles in the sense of Voorhees and Poggio do, in spite of find an approximate measure. Without leaving the granulometry tool we would accomplish this by defining efficient *number granulometries.*

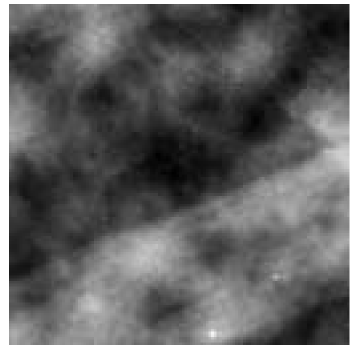
Adjacent to the previous ideas we have presented a general framework for texture discrimination based on the granulometric approach. We have shown some examples on how to construct granulometries. These enable us to estimate different attributes of local features, such as orientation, shape, size or contrast, in the framework of texton theory. For this purpose we have extended the concept of *full pattern spectrum*,[19] to an orientation-size descriptor. And we have used the classical granulometry as a tool to compute size density of particles. Finally we have exposed the problems of finding a good algebraic opening to measure the density of blobs contrast, although we have shown some works wich can help in defining such a transformation.



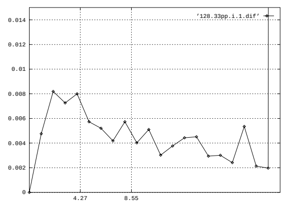
(a)



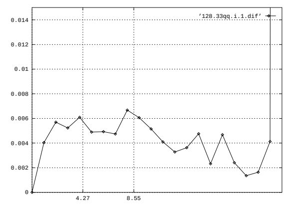
(b)



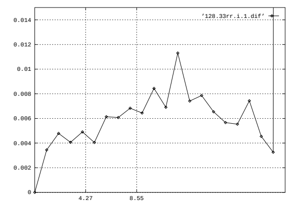
(c)



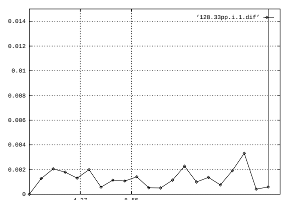
(a.1)



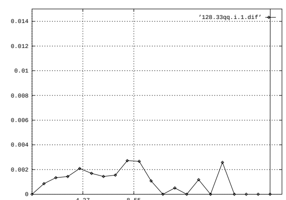
(b.1)



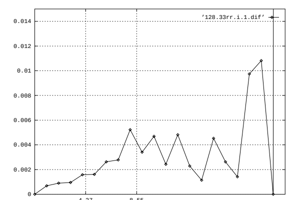
(c.1)



(a.2)



(b.2)



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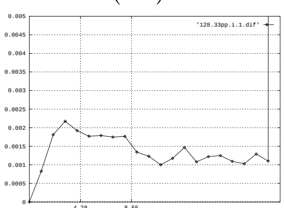
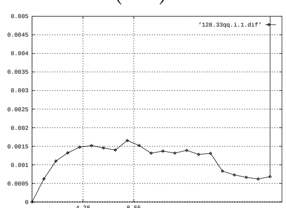
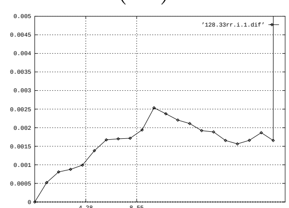


(a.3)



(b.3)



(c.3)

Figure 3: (a), (b) and (c) are radiographic images showing a textural pattern produced by rounded opacities of small, medium and large size, respectively. (a.1), (b.1) and (c.1) are the three corrresponding granulometric curves by a family of morphological openings. (a.2), (b.2) and (c.2) granulometric curves by a family of algebraic openings based on a reconstruction transformation. (a.3), (b.3) and (c.3) granulometric curves by a family of algebraic openings as supremum of various morphological openings

For a complete framework we can introduce the concept of local granulometric size distributions exposed by Dougherty[5] for complete segmentation purposes.

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